Laws of Motion

PHYSICS

For NEET / JEE

FORCE :

Force is a pull or push which generates or tends to generate motion in a body at rest, stops or tends to stop a body in motion, increases or decreases the magnitude of velocity of the moving body, changes or tends to change the shape of body.

Types of Forces :

There are basically three forces which are commonly encountered in mechanics.

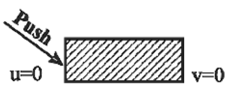
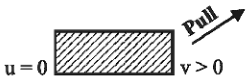
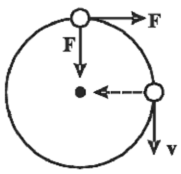
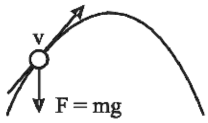
Field Forces :

These are the forces in which contact between two objects is not necessary. Gravitational force between two bodies and electrostatic force between two charges are two examples of field forces. Weight ($w = mg$) of a body comes in this category.

Contact Forces :

Two bodies in contact exert equal and opposite forces on each other. If the contact is frictionless, then contact force is perpendicular to the common surface and known as **normal reaction.**

If, however the objects are in rough contact and move (or have a tendency to move) relative to each other without losing contact, then **frictional force** arises which opposes such motion. Again each object exerts a frictional force on the other and the two forces are equal and opposite. This force is perpendicular to normal reaction. Thus, the contact force (F)

Body at Rest	
	If body remains at rest, force is trying to change the state of rest
	If body moves, force is changing the state of rest
Body In Motion	
 Magnitude increases Magnitude decreases (a)	When force is parallel or antiparallel to motion, it changes the magnitude and not the direction of motion.
When force is always perpendicular to motion.  (b)	Direction of motion only changes and not the magnitude and motion is uniform circular.
When force acts at an angle to the motion.  (c)	Both magnitude and direction of motion changes and motion is non-uniform circular, elliptic, parabolic or hyperbolic

between two objects is made up of two forces.

(a) Normal reaction (N)

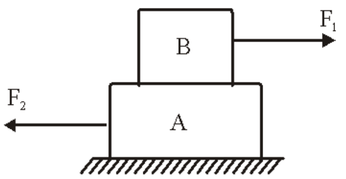
(b) Force of friction (f)

and since these two forces are mutually perpendicular.

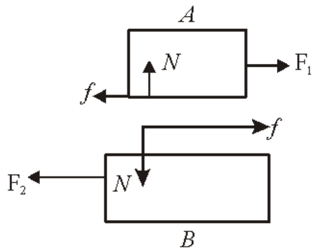
$$F = \sqrt{N^2 + f^2}$$

Consider two wooden blocks A and B being rubbed against each other.

In figure (below) A is being moved to the right while B is being moved leftward. In order to see more clearly which forces act on A and which on B, a second diagram is drawn showing a space between the blocks but they are still supposed to be in contact.



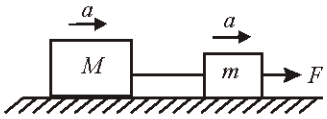
In figure (below) the two normal reactions each of magnitude N are perpendicular to the surface of contact between the blocks and the two frictional forces each of magnitude f act along that surface, each in a direction opposing the motion of the block upon which it acts.

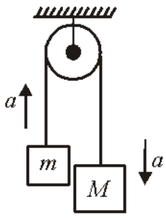


❖ Attachment to Another Body :

Tension (T) in a string and **spring force** ($F = kx$) come in this group. Regarding the tension and string, the following three points are important to remember.

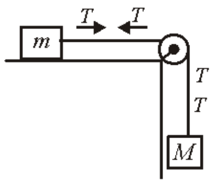
1. If a string is inextensible, the magnitude of acceleration of any number of masses connected through string is always same.
2. If a string is massless, the tension in it is same everywhere. However, if a string has a mass, tension at different points will be different.



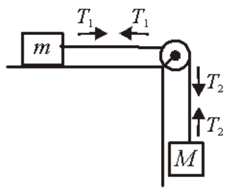


3. If there is a friction between string and pulley, tension is different on two sides of the pulley, but if there is no friction between pulley and string, tension will be same on both sides of the pulley.

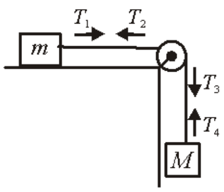
   Last two points can be understood in diagram follows :



String is massless and there is no friction between pulley and string



String is massless and there is friction between string and pulley



String is not massless and there is a friction between pulley and string

❖ Action of force on a body :

- Force is polar vector as it has a point of application.
- Dimension $[F] = [MLT^{-2}]$
- **Units :**
  - M.K.S. System $\rightarrow$ Kg m/s^2 (Newton)
  - C.G.S. System $\rightarrow$ gm cm/s^2 (Dyne)
  - [1 Newton $= 10^5$ Dyne]
  - F.P.S. System $\rightarrow$ 1b-ft/s^2 $\rightarrow$ Poundal
- When a force is applied by man (muscle) or machine there is an upper limit to it. This limit is different for different muscles or machines.

- When force is written without direction then positive force means attractive.
- $F_{nuclear} > F_{electromagnetic} > F_{gravitational}$
- According to dependence on position or motion force can be divided into following types :

➤ **Constant Force :**

Direction and magnitude of a force is constant. e.g. Mechanical force of constant magnitude produced by machines.

➤ **Time Dependent Force :**

Force depends on time. e.g. motion of charged particle in an alternating current field.

➤ **Position Dependent Force :**

Depends on the position of the body. e.g. Gravitational force $\left(F = \frac{G m_1 m_2}{r^2}\right)$ force between two charged particle $\left(F = \frac{q_1 q_2}{4\pi \in_0 r^2}\right)$

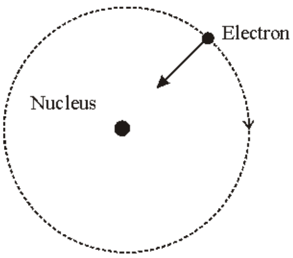
➤ **Velocity Dependent Force :**

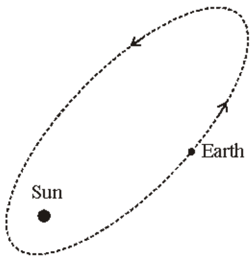
Depends on the velocity of the body. e.g. : viscous Force $(6\pi\eta r v)$, Force on a charged particle in a magnetic field $(qvB\sin\theta)$

Note In case of position dependent force is always directed towards or away from a fixed point and said to be central otherwise non central force.

➤ **e.g.:**

The motion of earth around the sun and motion of electron in an atom are examples of central force.





- **Internal and External force :**

When the force applying agent is inside the system, then force is internal.

It can't provide motion.

e.g. if you are sitting in a car and push the car, car doesn't move.

❖ Total internal force acting on a system is always zero.

❖ External force are those which originated beyond the system.

❖ The straight line along which force acts is called **line of action of the force**.

- **Closed system :**

A system of bodies on which no external force acts is called closed system. e.g. two bodies moving towards each other due to their electrostatic or gravitational force.

- **Concurrent force :**

When many forces act on a same point of a body are called concurrent force. It may be collinear (acting along same line) coplanar (in same plane). Forces act on different direction or plane.

- **Conservative or non conservative force :**

If under the action of a force the work done in a round trip is zero or the work is path independent, or there is no change in K.E., the force is said to be conservative otherwise non conservative.

Example *Conservative force* : Gravitational force, electrical force, elastic force and central forces are conservative force.

Non conservative force : Frictional force, viscous force.

- If number of forces are acting on a body, we find the resultant force using law of parallelogram of addition or we can resolve the forces as i.e.,

$$\vec{F_1} = F_{1x}\hat{i} + F_{1y}\hat{j} + F_{1z}\hat{k}$$

$$\vec{F_2} = F_{2x}\hat{i} + F_{2y}\hat{j} + F_{2z}\hat{k}$$

Similarly $\vec{F_3} = F_{3x}\hat{i} + F_{3y}\hat{j} + F_{3z}\hat{k}$

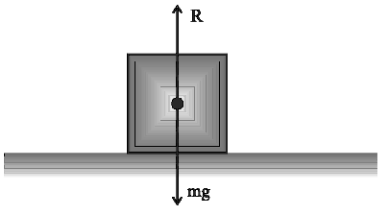
The resultant force

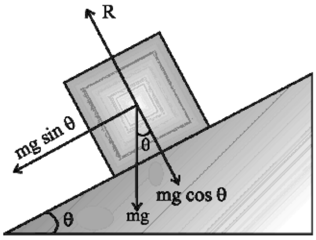
$$\vec{F} = [F_{1x} + F_{2x} + \cdots]\hat{i} + [F_{1y} + F_{2y} + \cdots]\hat{j} + [F_{1z} + F_{2z} + \cdots]\hat{k}$$

- **Common forces in mechanics:**

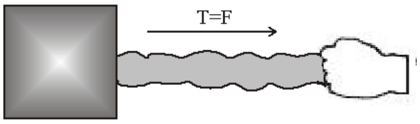
(i) Weight : Weight of an object is the force with which earth attracts it. It is also called the force of gravity or the gravitational force.

(ii) Reaction or Normal force : when a body is [placed on a rigid surface, the body experiences a force which is perpendicular to the surfaces in contact. This force is called **Normal force** or **Reaction.**

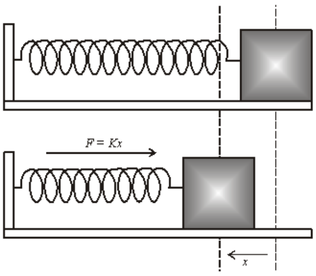




(iii) Tension : The force exerted by the end of taut string, rope or chain against pulling (applied) force is called the **tension.** the direction of tension is so as to pull the body away and from the body. Tension force is an electromagnetic force.



(iv) Spring force : Every spring resists any attempt to change its length. This resistive force increases with change in length. Spring force is given by $F = -Kx$; where x is the change in length and K is the spring constant (unit N/m)



Inertia:

The inherent property of the body with which it cannot change its state of rest or of uniform motion unless acted upon by an external force, is called inertia. This experience is enshrined in the Newton's first law of motion.

Newton's first law of motion :

An object at rest wants to remain at rest and an object in motion wants to move with uniform velocity until and unless an unbalanced (external) force is impressed on it.

Some Important points :

(1) If the forces be present but be balanced the object will not change their natural states. By balanced forces we mean that their vector sum is zero. Also, for balancing each other, the forces must act on the same body.

(2) The first law gives us the concept of inertia. It is also called the law of inertia.

(3) Rest and velocity are frame dependent. The force between objects depends upon attributes of objects.

(4) The first law of motion is valid only in certain frames. Such frames are called **inertial reference frames**. The frames accelerated or rotating as seen from an inertial frame are **'non-inertial'**. We state that all the three laws of motion are valid in **inertial** frames only.

Examples of First Law :

(1) If a body is thrown up from a uniformly moving train then it returns back to the same place.

(2) When a stationary vehicle suddenly moves then the passengers inside the vehicle fall backwards.

(3) When a carpet or a blanket is beaten with a stick then the dust particle is separated out from it.

(4) A coin on the cardboard placed on glass falls into the glass if it is given a sudden jerk.

(5) If a moving vehicle suddenly stops then the passengers inside the vehicle bend forward.

(6) The window glass pane is broken by a bullet from a gun.

Free Body Diagram:

A free body diagram (FBD) consists of a diagrammatic representation of a single body or a sub-system of bodies isolated from its surrounding showing all the forces acting on it.

In solving problems when several forces are involved, the following steps are recommended.

(a) First, fix up the system whose motion you want to consider to solve a given problem.

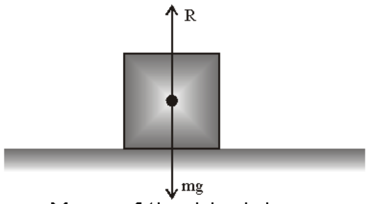
(b) Next, turn your attention to the objects which are the immediate environment of the system. These will exert forces on the system.

(c) Now make a separate diagram of the system alone, showing all the forces acting on the system, and the frame of reference. This is called a free-body diagram.

(d) Finally apply Newton's second law.

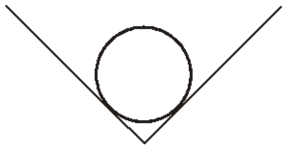
For example

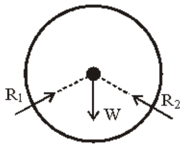
A block lying on a horizontal table



Mass of the block is m.

A cylinder of weight w is resting on a V-groove as shown in fig.. Draw its free body diagram.





FBD

Reaction in moving lift :

Let a man of mass m be in a lift.

Downward force of attraction acting on him = real weight = mg.

the upward reaction of the floor = apparent weight = R

The net upward force F = R – mg

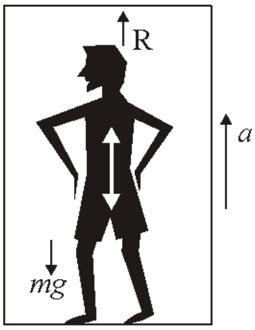
We can determine apparent weight R using F = ma for different types of lift's motion.

Case - I

The lift is moving up with an acceleration a (moving down with retardation a) :

Here the upward acceleration is a.

$\therefore R - mg = ma$



or, $R = mg + ma = m(g + a)$

$\therefore$ As $R > mg$ the man feels heavier.

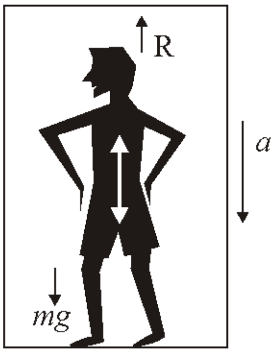
Case - II

The lift is coming down with an acceleration a (moving upward with retardation a) :

Here the downward acceleration = a

or upward acceleration = $-a$

$\therefore R - mg = -ma$



or, $R = m(g - a)$

Thus he feels lighter as

$m(g - a) < mg, (a < g)$

Case - III

The lift cable snaps and the lift with the man, due to gravitational pull, falls freely :

Here, the acceleration, $a = g$. Hence from (2)

$R = m(g - g) = 0$

As no reaction force acts on the man, he feels weightless. This apparent weightlessness is true for all freely falling bodies.

Case - IV

The lift is at rest or in uniform motion :

The value of a being zero R, the reaction forced = mg. Hence, apparent weight is same as the real or true weight.

Case - V

The lift falls with a downward acceleration, greater than g.

Here, $a > g$, from equation (2).

$R = m(g - a)$ or, $R = -m(a - g)$

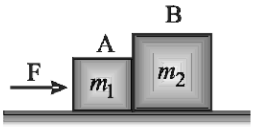
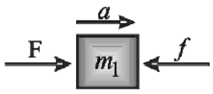
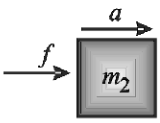
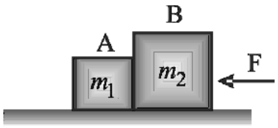
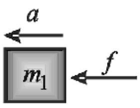
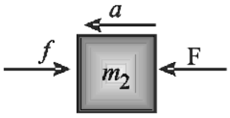
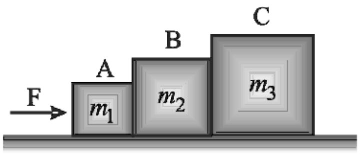
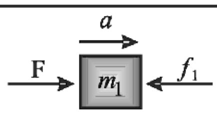
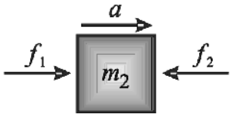
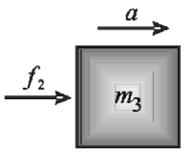
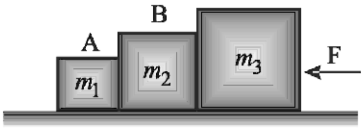
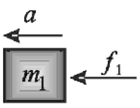
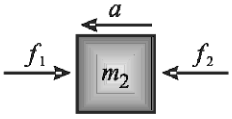
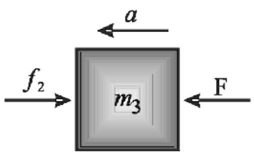
Negative sign signifies the reaction force R to be downward and therefore weight should be directed upward.

The man thus loses contact with the floor and will be hit the ceiling of the lift. This will be the state of any item lying on the floor of the lift. This condition was previously termed as super-weightlessness.

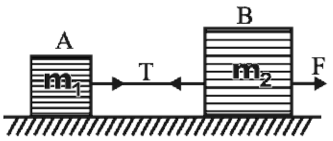
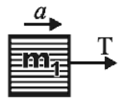
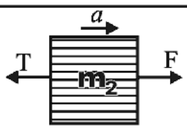
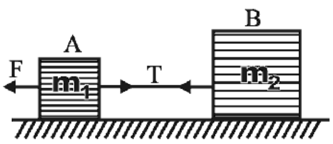
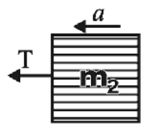
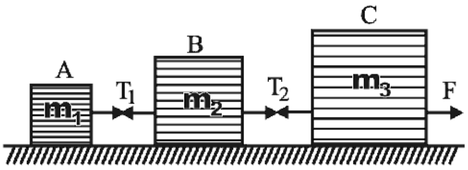
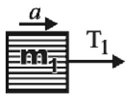
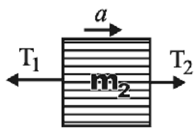
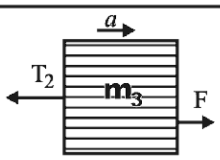
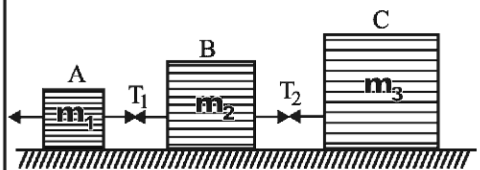
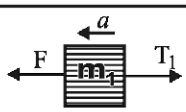
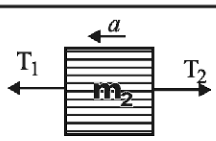
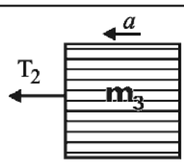
Constraint Equations :

These equations basically establish the relation between accelerations (or velocities) of different masses attached by string(s). Usually it is observed that the number of constraint equations are as many as the number of strings in the system under consideration.

Motion of Blocks In Contact

Condition	Free body diagram	Equation	Force and acceleration
		$F - f = m_1 a$	$a = \dfrac{F}{m_1 + m_2}$ $f = \dfrac{m_2 F}{m_1 + m_2}$
		$f = m_2 a - T_1$	
		$f = m_1 a$	$a = \dfrac{F}{m_1 + m_2}$ $f = \dfrac{m_1 F}{m_1 + m_2}$
		$F - f = m_2 a$	
		$F - f_1 = m_1 a$	$a = \dfrac{F}{m_1 + m_2 + m_3}$ $f_1 = \dfrac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$ $f_2 = \dfrac{m_3 F}{m_1 + m_2 + m_3}$
		$f_1 - f_2 = m_2 a$	
		$f_2 = m_3 a$	
		$f_1 = m_1 a$	$a = \dfrac{F}{m_1 + m_2 + m_3}$ $f_1 = \dfrac{m_1 F}{m_1 + m_2 + m_3}$ $f_2 = \dfrac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$
		$f_2 - f_1 = m_2 a$	
		$F - f_2 = m_3 a$	

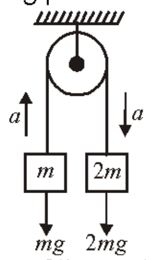
Motion of Blocks Connected by Massless String

Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - T = m_2 a$	$T = \frac{m_1 F}{m_1 + m_2}$
		$F - T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$T = m_2 a$	$T = \frac{m_2 F}{m_1 + m_2}$
		$T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_2 - T_1 = m_2 a$	$T_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$
		$F - T_2 = m_3 a$	$T_2 = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$
		$F - T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_1 - T_2 = m_2 a$	$T_1 = \frac{(m_2 + m_1)F}{m_1 + m_2 + m_3}$
		$T_2 = m_3 a$	$T_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$

Pulleys :

Now, let us see what is this pulling force method with the help of an example.

Suppose, two unequal masses m **and** $2m$ **are** attached to the ends of a light inextensible string which passes over a smooth massless pulley. We have to find the acceleration of the system. We can assume that the mass $2m$ is pulled downwards by a force equal to its weight, *i.e.*, $2mg$. Similarly, the mass m is being pulled by a force of mg downwards. Therefore, net pulling force on the system is 2mg – mg = mg and total mass being pulled is 2m + m = 3m.

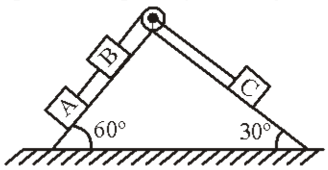


∴ Acceleration of the system is

$$a = \frac{\text{Net pulling force}}{\text{Total mass to be pulled}} = \frac{mg}{3m} = \frac{g}{3}$$

Concept Illustrator

In the below figure, mass of A, B and C are 1 kg, 3 kg and 2 kg, respectively. Find



(a) the acceleration of the system.

(b) tensions in the string.

Neglect friction. (g = 10 m/s²)

Sol. (a) In this case, net pulling force

$= m_A g \sin 60° + m_B g \sin 60° - m_C g \sin 30°$

$$= (1)(10)\frac{\sqrt{3}}{2} + (3)(10)\left(\frac{\sqrt{3}}{2}\right) - (2)(10)\left(\frac{1}{2}\right)$$

= 21.17 N

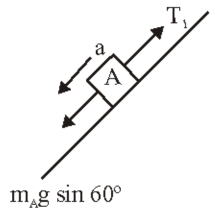
Total mass being pulled = 1 + 3 + 2 = 6 kg

∴ Acceleration of the system,

$$a = \frac{21.17}{6} = 3.53 \text{ m/s}^2$$

(b) For the tension in the string between A and B

FBD of A



$m_A g \sin 60° - T_1 = (m_A)(a)$

$\therefore T_1 = m_A g \sin 60° - m_A a$

$= m_A (g \sin 60° - a)$

$$\therefore T_1 = (1)\left(10 \times \frac{\sqrt{3}}{2} - 4.1\right)$$

= 5.13 N

For the tension in the string between B and C,

FBD of C

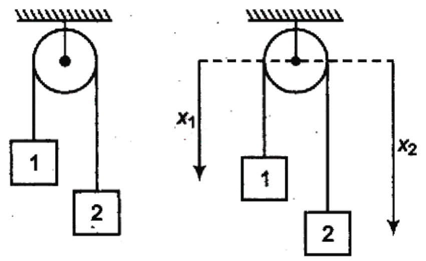
$T_2 - m_C g \sin 30° = m_C a$

$\therefore T_2 = m_C (a + g \sin 30°)$

$$\therefore T_2 = 2\left[3.53 + 10\left(\frac{1}{2}\right)\right] = 17.01 \text{ N}$$

Concept Illustrator

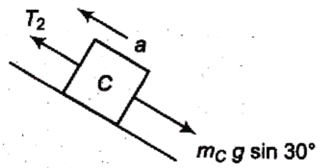
Using constraint method find the relation between accelerations of 1 and 2.



Sol. At any instant of time, let x_1 and x_2 be the displacements of 1 and 2 from a fixed line (shown dotted)

Then, $x_1 + x_2$ = constant

or $x_1 + x_2 = l$ (length of string)



Differentiating with respect to time, we have

$v_1 + v_2 = 0$

or $v_1 = -v_2$

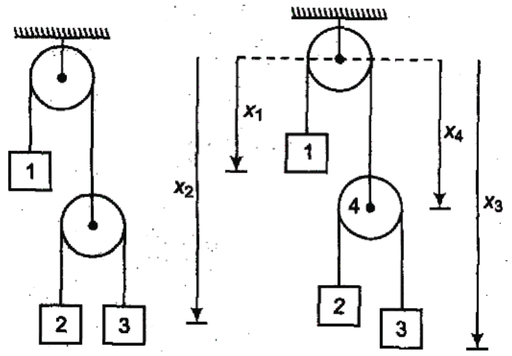
Again differentiating with respect to time, we get

$a_1 + a_2 = 0$ or $a_1 = -a_2$

This is the required relation between a_1 and a_2, i.e., accelerations of 1 and 2 are equal but in opposite directions.

Concept Illustrator

Find the constaraing relation between a_1, a_2 and a_3.



Sol. Points 1, 2, 3 and 4 are movable. Let their displacements from a fixed line be x_1, x_2, x_3 and x_4
we have

$x_1 + x_4 = l_1$ (length of first string) ··· (i)

and $(x_2 - x_4) + (x_3 - x_4) = l_2$
(length of second string)

or $x_2 + x_3 - 2x_4 = l_2$ ··· (ii)

On double, differentiating with respect to time, we get

$a_1 + a_4 = 0$ ··· (iii)

and $a_2 + a_3 - 2a_4 = 0$ ··· (iv)

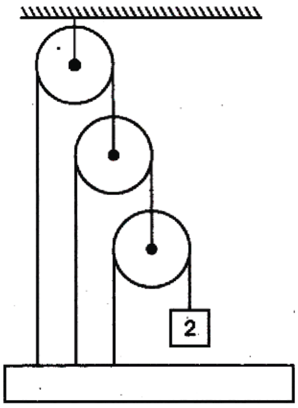
But since, $a_4 = -a_1$ [From eq. (iii)]

We have $a_2 + a_3 + 2a_1 = 0$

This is the required constaraint relation between a_1, a_2 and a_3.

Concept Illustrator

Using constraint equations find the relation between a_1 and a_2



Sol. Points 1, 2, 3 and 4 are movable. Let their displacements from a fixed line be x_1, x_2, x_3 and x_4.

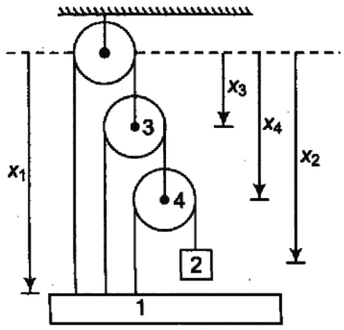
$x_1 + x_3 = l_1$

$(x_1 - x_3) + (x_4 - x_3) = l_2$

$(x_1 - x_4) + (x_2 - x_4) = l_2$

On double differentiating with respect to time, we will get following three constraint reations.

$a_1 + a_3 = 0$ ··· (i)



$a_1 + a_4 - 2a_3 = 0$ ··· (ii)

$a_1 + a_2 - 2a_4 = 0$ ··· (iii)

Solving eqs. (i), (ii) and (iii), we get

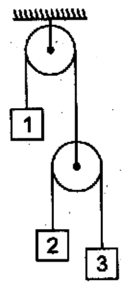
$a_2 = -7a_1$

which is the desired relation between a_1 and a_2.

Concept Illustrator

At certain moment of time, velocities of 1 and 2 both are 1m/s upwards. Find the velocity of 3 at that moment.

Sol. We know $a_2 + a_3 + 2a_1 = 0$



Similarly, we can find

$v_2 + v_3 + 2v_1 = 0$

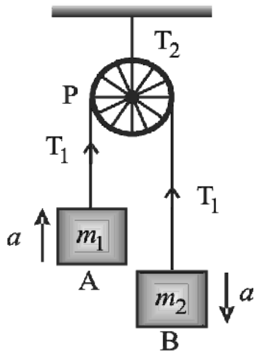
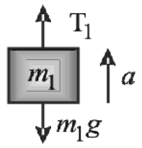
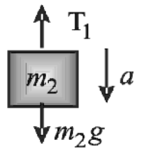
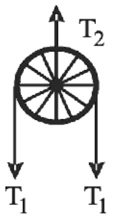
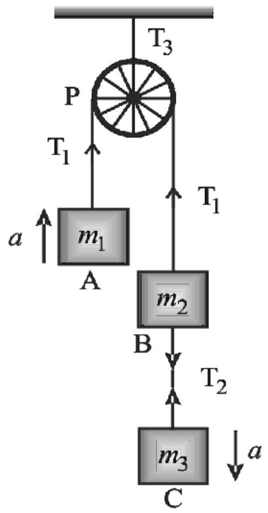
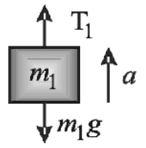
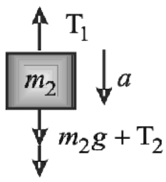
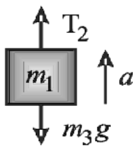
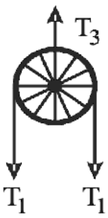
Taking upward direction as positive we ar given

$v_1 = v_2 = 1$ m/s

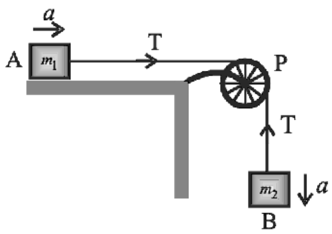
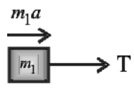
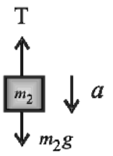
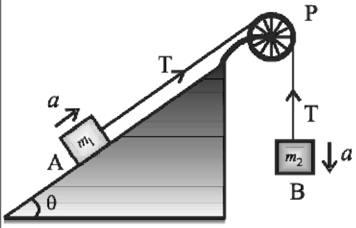
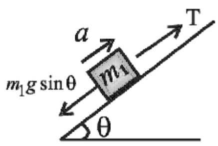
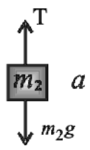
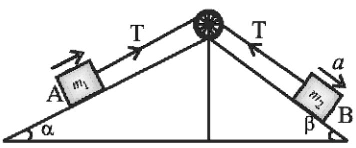
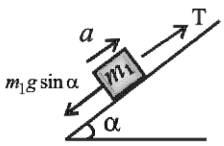
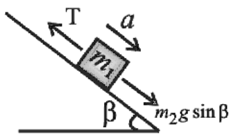
$\therefore$ $v_3 = -3$m/s

i.e., velocity of block 3 is 3 m/s (downwards)

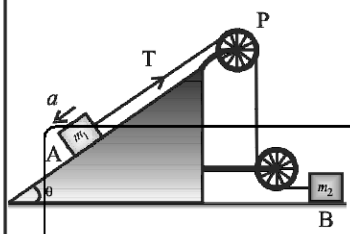
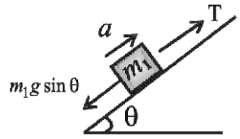
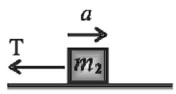
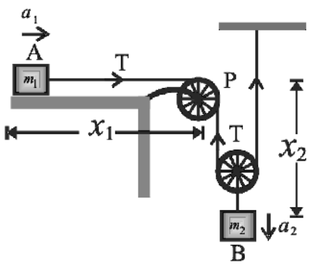
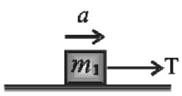
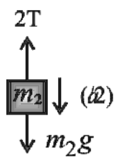
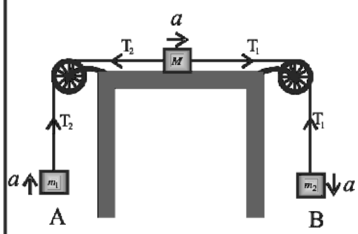
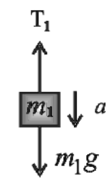
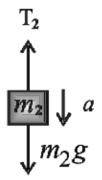
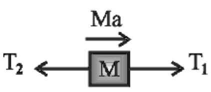
Motion of Connected Blocks Over A Pulley

Condition	Free body diagram	Equation	Tension and acceleration
		$m_1 a = T_1 \quad m_1 g$	$T_1 = \frac{2m_1 m_2}{m_1 + m_2} g$
		$m_2 a = m_2 g \quad T_1$	$T_2 = \frac{4m_1 m_2}{m_1 + m_2} g$
		$T_2 = 2T_1$	$a = \left[\frac{m_2 \quad m_1}{m_1 + m_2}\right] g$
		$m_1 a = T_1 \quad m_1 g$	$T_1 = \frac{2m_1 [m_2 + m_3]}{m_1 + m_2 + m_3} g$
		$m_2 a = m_2 g + T_2 \quad T_1$	$T_2 = \frac{2m_1 m_3}{m_1 + m_2 + m_3} g$
		$m_3 a = m_3 g \quad T_2$	$T_3 = \frac{4m_1 [m_2 + m_3]}{m_1 + m_2 + m_3} g$
		$T_3 = 2T_1$	$a = \frac{[(m_2 + m_3) \quad m_1] g}{m_1 + m_2 + m_3}$

Motion of Connected Blocks Over A Pulley

Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$	$a \frac{m_2}{m_1+m_2} g$
		$m_2 a = m_2 g - T$	$T = \frac{m_1 m_2}{m_1+m_2} g$
		$m_1 a = T - m_1 g \sin\theta$	$a = \left[\frac{m_2 - m_1 \sin\theta}{m_1+m_2}\right] g$
		$m_2 a = m_2 g - T$	$T = \frac{m_1 m_2 (1+\sin\theta)}{m_1+m_2} g$
		$T - m_1 g \sin\alpha = m_1 a$	$a = \frac{(m_2 \sin\beta - m_1 \sin\alpha)}{m_1+m_2} g$
		$m_2 a = m_2 g \sin\beta - T$	$T = \frac{m_1 m_2 (\sin\alpha + \sin\beta)}{m_1+m_2} g$

Motion of Connected Blocks Over A Pulley

		$m_1 g \sin\theta - T = m_1 a$	$a = \frac{m_1 g \sin\theta}{m_1 + m_2}$
		$T = m_2 a$	$T = \frac{2m_1 m_2 \sin\theta}{4m_1 + m_2}$
		$T = m_1 a$	$a_1 = a = \frac{2m_2 g}{4m_1 + m_2}$ $a_2 = \frac{m_2 g}{4m_1 + m_2}$ $T = \frac{2m_1 m_2 g}{4m_1 + m_2}$
		$m_2 \frac{a}{2} = m_2 g - 2T$	
		$m_1 a = m_1 g - T_1$	$a = \frac{(m_1 \quad m_2)}{[m_1 + m_2 + M]} g$
		$m_2 a = T_2 - m_2 g$	$T_1 = \frac{m_1(2m_2 + M)}{[m_1 + m_2 + M]} g$
		$T_1 - T_2 = Ma$	$T_2 = \frac{m_2(2m_2 + M)}{[m_1 + m_2 + M]} g$

Momentum

The dynamical property that arises from the combined effect of mass and velocity of a body is called its momentum.
The product of mass and velocity to body is known as **momentum.**

We denote momentum by $\vec{P}$:

$$\vec{P} = m\vec{v}$$

Important points :
Momentum is a vector quantity.
Dimensional formula : [MLT^{-1}]

S.I. Unit : kg ms^{-1}

The Newton's second law motion

The time of change of momentum of a body is directly proportional to the unbalanced external force and the change takes place in the direction of the unbalanced external force.

If $\vec{P}$ be the momentum of a body of mass *m* moving with velocity $\vec{v}$ at a time *t*, the rate of change of its momentum is $\frac{d\vec{p}}{dt}$.

Let us denote by $\vec{F}$ the unbalanced force (i.e., net force) acting on the body at this moment. Then the second law of motion states that

$$\frac{d\vec{p}}{dt} \propto \vec{F}$$

and $d\vec{p}$ is in the direction of $\vec{F}$. Expressing it using equality sign needs a constant of proportionality *k*. That is

$$\frac{d\vec{p}}{dt} = k\vec{F}$$

As $\vec{F}$ and $d\vec{p}$ are co-ordectional, *k* is positive.
The mass *m* of a body is constant as the particle moves at speeds lower than the speed of light. According Einstein that at higher speed *v*, the mass varies with speed *v* as

$$m = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

where m_0 is the mass speed is low $(v << c)$.
We shall consider bodies moving at such non-relativistic speed. Their mass will be taken as constant. In this special case.

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

where $\vec{a}$ is the acceleration, $\frac{d\vec{v}}{dt}$.
Then, the second law of motion may be recast for bodies of constant mass as

$$m\vec{a} = k\vec{F}.$$

We define unit force as that force which produces unit acceleration in a body of unit mass.
Then *k* =1. Such a choice of unit force allows us to write the second law of motion as

$$\vec{F} = m\vec{a}$$

Here, $\vec{F}$ is the vector sum of all the forces acting on the body and $\vec{a}$ is its vector acceleration.

Impulse of Force
Definition : For a constant force that acts on a body for a short interval of time, the product of the force and the time is the impulse of the force.
Let us consider a body of mass m being acted upon by a force *F* for time *t*.
$\therefore$ Impulse of force = *F* × *t*
Now, because of the force applied on the body, the velocity of the body changes from u to v and its acceleration is *a* .
Therefore, impulse of force = *F*×*t* = *ma* × *t*

$= m\frac{v-u}{t} \times t$ = m(v – u)

= (mv – mu) .... (1)
= change in momentum
In other words, the change in momentum of a body is the impulse.
Force and impulse are both vector quantities. Therefore, equation (1) is a vector quation and the direction of impulse is same as that of the force itself.
Unit of impulse : Unit of impulse of force = unit of force × unit of time

CGS system	FPS system	SI
dyn.s	poundal.s	N.s

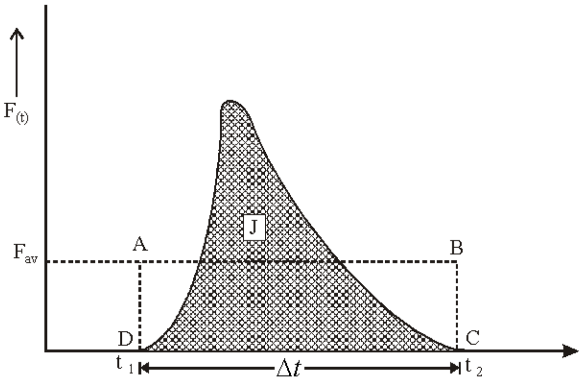
Dimensional formula : [MLT^{-1}]

Impulsive Force :

If a large force acts on a body or particle for a small time the product of force with time is defined as impulsive force.

i.e., $\vec{J} = \int_{t_1}^{t_2} \vec{F}\, dt = F_{av} \int_{t_1}^{t_2} dt = F_{av} \Delta t$

F_{av} is the average magnitude of force and Δt is the duration for which force acts.



The value of F_{av} must be taken show that the area within the rectangle of fig. is equal to the area under the F(t) curve. i.e., $F_{av} = \frac{J}{\Delta t}$.

It is a vector quantity.

Dimensional formula : [MLT^{-2}]

Unit : Newton in S.I. System

Important :

(i) According to Newton's second law of motion we get

$\vec{F} = \frac{d\vec{p}}{dt}$ i.e., $\int_{t_1}^{t_2} \vec{F}\, dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p}$

From $\vec{J} = \int_{t_1}^{t_2} \vec{F}\, dt$ we can reduced

$\vec{J} = \vec{p}_2 - \vec{p}_1 = \Delta \vec{p}$

i.e., the action of impulse is to change the momentum of a body or particle and the impulse of a force is equal to the change in momentum. This ststement is known as impulse-momentum theorem.

Example

Hitting, Kicking, Catching, Collision etc. are the example of impulse.

We know, $J = \int F\, dt = F_{av} \Delta t = \Delta p = \text{constant}$

Then $F_{av} = \frac{\text{constant}}{\Delta t}$, i.e., if time of contact Δt is increased, average force is decreased and *vice-versa*.

Note

- In hitting or kicking a ball if we decreased the time of contact so that large force act on the ball producing greater acceleration.
- In the same way –
  In catching a ball a player by drawing his hands backwards increases the time of contact because the lesser force act on his hands and his hands are saved from greeting hurt.

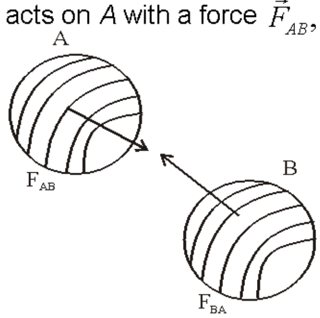
Newton's Third Laws of Motion

Traditional Statement : For every action there is an equal and opposite reaction.

Modern Version :

When two bodies exert mutual forces on one another, the two actions are always equal in magnitude but opposite in direction.

Let A act on B by a force $\vec{F}_{BA}$, then *B* also acts on *A* with a force $\vec{F}_{AB}$, such that



$\vec{F}_{AB}, = \vec{F}_{BA}$ (all the time)

This is the content of the third law of motion.

Features

1. A single isolated force is physically impossible. Whenever one force acts on a body it gives rise to another force called reaction. *i.e.* Total internal force in an isolated system is always zero.
2. Action and reaction are mutual forces and hence have the same nature. Action and reaction act at the same time. There is no "cause-effect" relationship between them.
3. The lines of action of forces of action and reaction may either be the same or parallel. If action and reaction act along the same line we say that third law is holding good in *strong form*. If the lines of action and reaction are parallel,

we say that the third law is holding good but in *weak form*.

4. The law is valid only in an inertial frame. If a frame is not inertial, the observer may notice a force-like quantity but not obeying the third law of motion. Such force-like quantities have *no bodily origin* and are called *pseudo forces*.
5. The impulse of action is equal in magnitude and opposite in direction to the impulse of reaction. If impulse of action be $\vec{j}_{AB}$ and impulse of reaction be $\vec{j}_{BA}$, then $\vec{j}_{BA} = \vec{j}_{AB}$

Example

(a) When a bullet is fired from a gun (action), the gun recoils due to reaction.

(b) In swimming or boating we push the water backward (action) and water pushes us or boat forward (reaction) giving rise to the required motion.

(c) In moving on earth we push the earth back (action), and as reaction to it, the earth pushes us in the forward direction. However, our mass is much smaller than that of earth, so acceleration is much greater.

(d) *The working of rocket or jet is also based on Newton's III law* as due to ejection of burnt gas at high speed (action), the rocket or jet moves in opposite direction reaction.

(e) In case of an orbital motion of satellite, the force exerted by gravitational pull of earth on the satellite is action and the force which the satellite exerts on the earth is reaction. In accordance with Newton's III law both are equal and opposite. However, as mass of earth is much greater than that of satellite, its acceleration is much lesser than that of satellite (acceleration. = force / mass)

- **Concurrent force**

When many force act on a same point of a body are called concurrent force. It may be collinear (acting along same line) coplanar (in same plane). Forces act on different direction or plane.

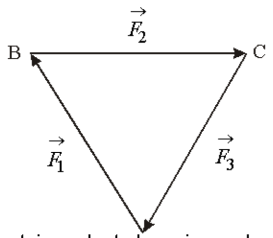
- **Equilibrium of concurrent force**

- The necessary condition for the equilibrium of a body under the action of concurrent force is that the vector sum of all the force acting on the body must be zero.

i.e., $\sum \vec{F}_{net} = 0$

or $\sum F_x = 0; \sum F_y = 0;, \sum F_z = 0$

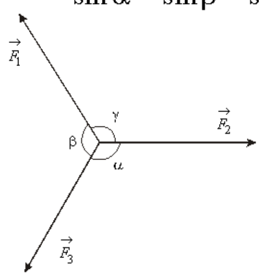
- Three concurrent forces will be in equilibrium, if they can be represented completely by three



sides of a triangle taken in order.

- **Lami's Theorem :** For three concurrent forces

in equilibrium $\dfrac{F_1}{\sin\alpha} = \dfrac{F_2}{\sin\beta} = \dfrac{F_3}{\sin\gamma}$



FRICTION

If we slide or try to slide a body over a surface the motion is resisted by a bonding between the body and the surface. This resistance is represented by a single force and is called friction.

- The force of friction is parallel to the surface and opposite to the direction of intended motion.
- If a body is resting on a horizontal surface and no force is applied, the forces acting are weight and reaction which balance each other. The force of friction is zero (and not μR). So if it is assumed to act on the body, the body will not be in equilibrium and the direction of force of friction cannot be decided, i.e., if a body is at rest and no pulling force is acting on it, force of gravitation on it is zero.

- **Friction is of two types.**

a. Static b. Kinetic

STATIC FRICTION

It exists between the two surfaces when there is tendency of relative motion but no relative motion along the two contact surface.

For example consider a table inside a room; when we gently push the table with a finger, the table does not move. This means that the table has a tendency to move in the direction of applied force but does not move as there exists static frictional force acting in the opposite direction of the applied force.

Direction of static friction :

The static friction on an object is opposite to its impending motion relative to the surface.

Following steps should be followed in determining the direction of static friction on an object.

(i) Draw the free body diagram with respect to the other object on which it is kept.

(ii) Include force also if contact surface is accelerating.

(iii) Decide the resultant force and the component parallel to the surface of this resultant force.

(iv) The direction of static friction is opposite to the above component of resultant force.

Note The static friction is involved when there is no relative motion between two surfaces.

Limiting Friction : Maximum opposing force when there is tendency of relative motion. It is equal to driving force during static state.

Laws of Limiting static friction :

(1) It depends upon nature of two surfaces in contact.

(2) It is proportional to the normal reaction between two surfaces.

$F_S \propto R \qquad \therefore F_S = \mu_S R$

R = normal reaction

μ_S = coefficient of static friction. It has no unit and no dimensions.

(3) Limiting friction does not depend upon area of contact between two surfaces.

Kinetic Friction

Kinetic friction exists between two contact surfaces only when there is **relative motion** between the two contact surfaces. It stops acting when relative motion between two surfaces ceases.

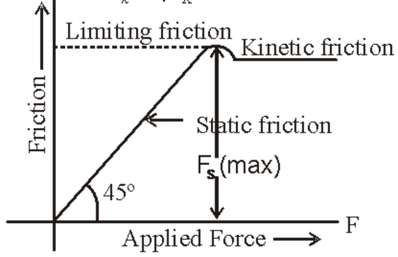
Direction of kinetic friction :

It is opposite to the relative motion of the object with respect to the other object in contact considered.

Note Its direction is not opposite to the force applied it is opposite to the relative motion of the body considered which is in contact with the other surface.

Laws of kinetic friction (i) It is constant in magnitude for small velocities. (ii) It does not depend upon area of contact. (iii) It is proportional to the normal reaction.

$F_K \propto R \qquad \therefore F_k = \mu_K R$



Rolling friction : When a body (say well) rolls on a surface the resistance offered by the surface is called rolling friction. In rolling the surfaces at contact do not rub each other. The velocity of the point of contact with respect to the surface remains zero all the time although the centre of the wheel moves forward. The rolling friction is negligible in comparison to static or kinetic friction which may be present simultaneously, i.e.,

μ_k = coefficient of kinetic friction.

μ_{sl} = coefficient of sliding, μ_r = coefficient of rolling friction.

$\mu_r < \mu_{sl} < \mu_s$

- Friction is a non-conservative force, i.e., work done against friction is path dependent. In its presence mechanical energy is not conserved as it converts, energy of motion (i.e., kinetic energy) of a body into heat. Thus, friction reduces efficiency of a machine. Lubrication reduces friction as it prevents interlocking of elevations and depressions.

Note : Misconception : Friction always oppose the motion.

- **Exception :**

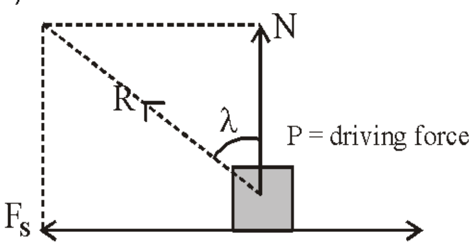
(i) In moving, a person or vehicle pushes the ground backwards (action) and the rough surface of ground reacts and exerts a forward force due to friction which causes the motion. If there had been no friction there will be slipping and no motion.

(ii) In cycling, the rear wheel moves by the force communicated to it by pedaling while front wheel moves by itself. So, when pedaling a bicycle, the force exerted by rear wheel on ground makes force of friction act on it in the forward direction (like walking). Front wheel moving by itself experience force of friction in backward direction (like rolling of a ball).

- Without friction motion can't be started, stopped or transferred from one body to other

ANGLE OF FRICTION

The angle made by the resultant of frictional force (F_S) and the normal reaction with the direction of the normal reaction (figure shows this).



Angle of friction

N = Normal reaction.

F_S = frictional force

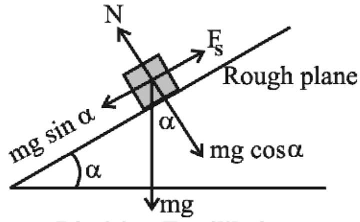
R = resultant of N and F_S.

λ angle of friction = $\tan^{-1}(F_S/N) = \tan^{-1}\mu_S$.

The coefficient of limiting friction is thus equal to the tangent of the angle of friction.

- **Angle of Repose :**

It is minimum angle of inclination of a rough inclined plane with the horizontal so that the body placed on it is in limiting equilibrium.



Limiting Equilibrium

α = angle of repose

mg = weight of the body, F_S = limiting friction = $mg \sin\alpha$

N = normal reaction = $mg \cos\alpha$

$$\mu_S = \frac{F_S}{N} = \frac{mg\sin\alpha}{mg\cos\alpha} = \tan\alpha$$

α = angle of repose = angle of friction λ

Acceleration of a body moving down an inclined plane when the angle of inclination of the plane is greater than angle of repose the body moves down with acceleration f.

θ = angle of inclination of the plane.

$\therefore \theta > \alpha$ then f = acceleration down; m = mass of the body; μ_k = coefficient of kinetic friction.

$$mf = mg\sin\theta - F_S = mg\sin\theta - \mu_k N$$

$$= mg\sin\theta - \mu_k\, mg\cos\theta$$

$$f = (g\sin\theta - \mu_k\cos\theta)/m$$

Work done during motion : In static state no work is done.

During motion upward the plane – w = work

$W = (mg\sin\theta + F) \times$ displacement along the plane

- **Advantage of friction :**

(1) It helps us walk, run on ground. It is easier to walk on rough ground than on snow, ice or slippery ground

(2) Helps to hold things in hands, write on black board, on paper

(3) Helps to tie knots of strings or ropes

(4) Brakes work due to friction

(5) Treading of tyres in done to increase friction

(6) Helps to transmit power from motors and engines to other machines

Disadvantages of friction :

(1) Causes unnecessary wear and tear of moving parts

(2) Friction produces heat due to motion of parts of engines

(3) Loss of energy and fuel due to friction

Lubrication :

If we put some 'grease' between surface in contact, the sliding friction is highly reduced. Such materials which when placed between sliding surfaces reduce the sliding friction significantly are known as Lubricants. Putting lubricants for this purpose is lubrication. Some lubricants are grease, oil, etc.

Why does friction reduce upon lubrication?

This is so because a thin layer of lubricant fluid comes between two solid surfaces trying to come in contact. As a result, the solid-fluid friction and fluid-fluid friction comes into play. These are very weak, thus reducing the friction.

Dynamics of circular motion :

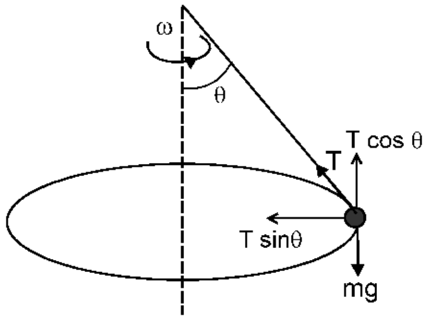
In circular motion or motion along any curved path Newton's law is applied in two perpendicular directions one along the tangent and other perpendicular to it . i.e. towards centre. The component of net force along the centre is called **centripetal force**. The component of net force along the tangent is called **tangential force**.

tangential force $(F_t) = Ma_t = M\dfrac{dv}{dt} = M\alpha r$

centripetal force $(F_c) = m\omega^2 r = \dfrac{mv^2}{r}$

Circular Motion in Horizontal Plane :

A ball of mass m attached to a light and inextensible string rotates in a horizontal circle of radius r with an angular speed ω about the vertical. If we draw the force diagram of the ball. We can easily see that the component of tension force along the centre gives the centripetal force and component of tension along vertical balances the gravitation force.



FBD of ball w.r.t. ground

Circular Turning on roads :

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways.

1. **By friction only**
2. **By banking of roads only.**
3. **By friction and banking of roads both.**

In real life the necessary centripetal force is provided by friction and banking of roads both. Now let us write equations of motion in each of the three cases separately and see what are the constant in each case.

1. **By Friction Only :**

Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction f acting towards center

Thus, $f = \dfrac{mv^2}{r}$

Further, limiting value of f is μN

or $f_L = \mu N = \mu mg \qquad (N = mg)$

Therefore, for a safe turn without sliding

$\dfrac{mv^2}{r} \le f_L$ or $\dfrac{mv^2}{r} \le \mu mg$

or, $\mu \ge \dfrac{v^2}{rg}$ or $v \le \sqrt{\mu rg}$

Here, two situations may arise. If μ and r are known to us, the speed of the vehicle should not exceed $\sqrt{\mu rg}$ and if v and r are known to us, the coefficient of friction should be greater than $\frac{v^2}{rg}$.

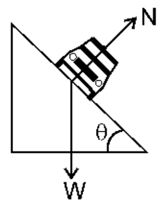
2. By Banking of Roads Only :

Friction is not always reliable at circular turns if high speeds and sharp turns are involved. to avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.

Applying Newton's second law along the radius and the first law in the vertical direction.

$$N \sin\theta = \frac{mv^2}{r}$$

or $N \cos\theta = mg$

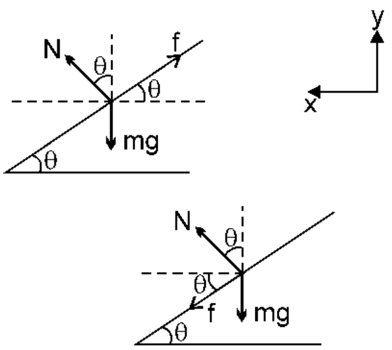


from these two equations, we get $\tan\theta = \frac{v^2}{rg}$

or, $v = \sqrt{rg\tan\theta}$

3. Both By Friction and Banking of Road:

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction.



The direction of second force, i.e., normal reaction N is also fixed (perpendicular ro road) while the direction of the third force i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_L = \mu N$). So the magnitude of normal reaction N and directions plus magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the center.

Of these m and r are also constant. Therefore, magnitude of N and directions plus magnitude of friction mainly depends on the speed of the vehicle v. Thus, situation varies from problem to problem. Even though we can see that :

(i) Friction f will be outwards if the vehicle is at rest v = 0. Because in that case the component weight mg sinθ is balanced by f.

(ii) Friction f will be inwards if $v > \sqrt{rg\tan\theta}$

(iii) Friction f will be outwards if $v < \sqrt{rg\tan\theta}$ and

(iv) Friction f will be zero if $v = \sqrt{rg\tan\theta}$

Note :

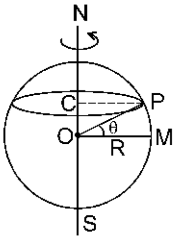
(i) The expression $\tan\theta = \frac{v^2}{rg}$ also gives the angle of banking for an aircraft, i.e., the angle through which it should tilt while negotiating a curve, to avoid deviation from the circular path.

(ii) The expression $\tan\theta = \frac{v^2}{rg}$ also gives the angle at which a cyclist should lean inward, when rounding a corner. In this case, θ is the angle which the cyclist must make with the vertical.

Effect of Earths Rotation on Apparent weight :

The earth rotates about its axis at an angular speed of one revolution per 24 hours. The line joining the north and the south poles is the axis of rotation.

Every point on the earth moves in a circle. A point at equator moves in a circle of radius equal to the radius of the earth and the centre of the circle is same as the centre of the earth. For any other point on the earth, the circle of rotation is smaller than this. Consider a place P on the earth (figure).

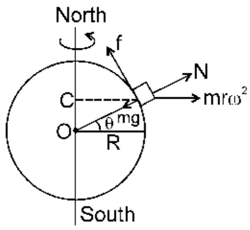


Drop a perpendicular PC from P to the axis SN. The place P rotates in a circle with the centre at C. The radius of this circle is CP. The angle between the line OM and the radius OP through P is called the latitude of the place P. We have

CP = OP cosθ or, r = R cosθ

where R is the radius of the earth.

If we work from the frame of reference of the earth, we shall have to assume



the existence of pseudo force. In particular, a centrifugal force mw^2r has to be assumed on any particle of mass m placed at P.

If we consider a block of mass m at point P then this block is at rest with respect to earth. If resolve the forces along and perpendicular the centre of earth then

$N + mr\omega^2 \cos\theta = mg \Rightarrow N = mg - mr\omega^2 \cos\theta \Rightarrow N = mg - mR\omega^2 \cos^2\theta$

Tips & Tricks

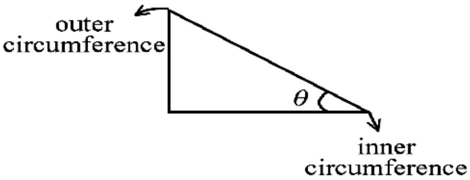
- Inertia is proportional to mass of the body.
- Force cause acceleration.
- A system or a body is said to be in equilibrium, when the net force acting on it is zero.
- A single isolated force cannot exist.
- 1gf = 980 dyne and 1kgf = 9.8 N
- Apparent weight of a freely falling body = ZERO, (state of weightlessness).
- For an isolated system (on which no external force acts), the total momentum remains conserved (Law of conservation of momentum).
- The chage in momentum of a body depends on the magnitude and direction of the applied force and the period of time over which it is applied i.e. it depends on its impulse.
- Recoil velocity of the gun is $\vec{V} = \frac{-m}{M}\vec{v}$

  Where m = mass of bullet, M = mass of gun and $\vec{v}$ = muzzle velocity of bullet.
- The rocket pushes itself forwards by pushing the jet of exhaust pases backwards.
- Upthrust on the rocket $= u \times \frac{dm}{dt}$
- Initial thrust on rocket = m(g+a), where a is the acceleration of the rocket.
- Upward acceleration of rocket $= \frac{u}{m} \times \frac{dm}{dt}$
- Newton's third law is applicable whether the bodies are at rest or in motion.
- Rolling friction is much less than the sliding friction. This knowledge was used by man to invent the wheels.
- The friction between two surfaces increases (rather than to decrease), when the surfaces are made highly smooth.
- The non-inertial character of the earth is evident from the fact that a falling object does not fall straight down but slightly deflects to the east due to coryolis (pseudo) force.
- When a body is moving with a linear velocity v around a circular path of radius r, then an acceleration known as centripetal acceleration acts on that body which is directed radially towards the axis of rotation of the body.

  centripetal acceleration $\vec{a}_c = \frac{v^2}{r}$ and

  centripetal force $\vec{F}_c = \frac{mv^2}{r} = mr\omega^2$

❖ For a body rotating in a circular path the necessary centripetal force is provided by making the path in such a way that the outer circumference of the path is at a higher height than the inner circumference of the path. This process is known as the banking of the curve.



The angle θ is known as the angle of banking

$\tan\theta = \frac{v^2}{rg}$, where v = linear velocity of the body

r = radius of curvature of the body

g = acceleration due to gravity.

❖ If a body is moving with constant angular velocity then the angular momentum, energy and speed remains constant for that body.

❖ A vehicle will slip down a banked road if $\frac{v^2}{r} > \mu g$, where

μ = coefficient of friction.

If there is no banking of the road, then the safe velocity of the vehicle is $v = \sqrt{\mu rg}$

For a banked road, the safe velocity is $v = \sqrt{\mu rg \cot\theta}$

For a plane road, the safest velocity that the vehicle will not topple over is $\frac{v^2 h}{r} < ag$

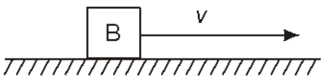
where 2a = distance between the two tyres.

h = the height of the centre of mass from the road.

The maximum velocity in such case is $v = \sqrt{\frac{agr}{h}}$

Solved Example

1. A block *B* is pushed momentarily along a horizontal surface with an initial velocity *v*. If μ is the coefficient of sliding friction between *B* and the surface, block *B* will come to rest after a time



a) $\frac{v}{g\mu}$ b) $\frac{g\mu}{v}$

c) $\frac{g}{v}$ d) $\frac{v}{g}$

Sol.: **(a)** Retardation due to friction = $-\mu g$

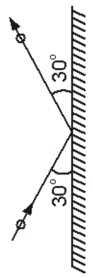
Initial velocity = *v*

Now using *v* = *u* + *at*

Final velocity is zero

$$\Rightarrow 0 = v - \mu g t \Rightarrow t = \frac{v}{\mu g}$$

2. A 0.5 kg ball moving with a speed of $12ms^{-1}$ strikes a hard wall at an angle of 30° with the wall. It is reflected with the same speed and at the same angle. If the ball is in contact with the wall for 0.25 s, the average force acting on the wall is



a) 48 N b) 24 N

c) 12 N d) 96 N

Sol.: **(b)**

$$\Delta p = OB\sin 30^\circ - (-OA\sin 30^\circ)$$

$$= mv\sin 30^\circ - (-mv\sin 30^\circ)$$

$$= 2mv\sin 30^\circ$$

Its time rate will appear in the form of average force acting on the wall.

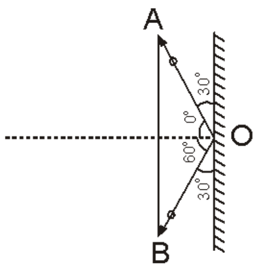
$\therefore \quad F \times t = 2mv\sin 30^\circ$

or $\quad F = \dfrac{2mv\sin 30^\circ}{t}$

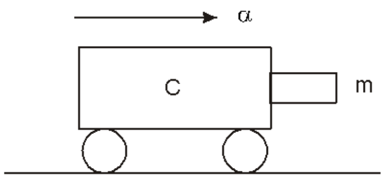
Given, $m = 0.5$ kg, $v = 12ms^{-1}$, $t = 0.25$ s

$\theta = 30^\circ$

Hence, $F = \dfrac{2 \times 0.5 \times 12\sin 30^\circ}{0.25}$ = 24 N



3. A block of mass m is in contact with the cart C as shown in the figure.



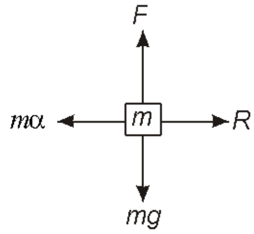
The coefficient of static friction between the block and the cart is μ. The acceleration α of the cart that will prevent the block from falling satisfies

a) $\alpha > \dfrac{mg}{\mu}$ b) $\alpha > \dfrac{g}{\mu m}$

c) $\alpha \geq \dfrac{g}{\mu}$ d) $\alpha < \dfrac{g}{\mu}$

Sol.: (c)

When cart moves with some acceleration towards right then a pseudo force (*ma*) acts on block towards left.



This force $(m\alpha)$ is action force by a block on cart. Now, block will remain static with respect to cart, if frictional force

$\mu R \geq mg$.

$\Rightarrow \quad \mu m\alpha \geq mg \qquad (R = m\alpha)$

$\Rightarrow \quad \alpha \geq \dfrac{g}{\mu}$

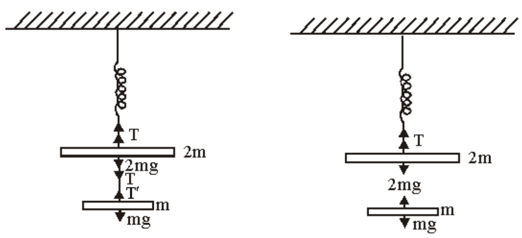
4. The string between blocks of mass m and $2m$ is massless and inextensible. The system is suspended by a massless spring as shown. If the string is cut find the magnitudes of accelerations of mass $2m$ and m (immediately after cutting)

(a) $g, \ g$ (b) $g, \dfrac{g}{2}$

(c) $\dfrac{g}{2}, g$ (d) $\dfrac{g}{2}, \dfrac{g}{2}$

Sol. By equilibrium of mass m,

$T' = mg$ ....(i)



By equilibrium of $2m$, $T = 2mg - T'$

From (i) and (ii), $T = 2mg - mg = mg$

When the string is cut :

For mass m : $F_{net} = ma_m$

$\Rightarrow mg = ma_m \Rightarrow a_m = g$

For mass $2m$:

$F_{net} = 2ma_{2m}$

$\Rightarrow 2mg - T = 2ma_{2m}$

$\Rightarrow 2mg - mg = 2ma_{2m}$

$\Rightarrow a_{2m} = \frac{g}{2}$

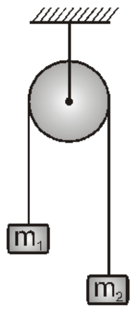
Hence choice is (c).

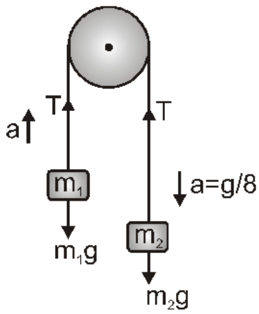
5. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is g/8, then the ratio of the masses is :

(a) 8 : 1 (b) 9 : 7

(c) 4 : 3 (d) 5 : 3

Sol. Free body diagram of system





Considering $m_2 > m_1$

$m_2g - T = m_2a$ ....(i)

$T - m_1g = m_1a$ ....(ii)

$a = \frac{(m_2 - m_1)}{m_1 + m_2} g$

Given, a = g/8

$\Rightarrow \frac{g}{8} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$

$m_1 + m_2 = 8(m_2 - m_1)$

$7m_2 = 4m_1$

$\frac{m_2}{m_1} = \frac{9}{7}$

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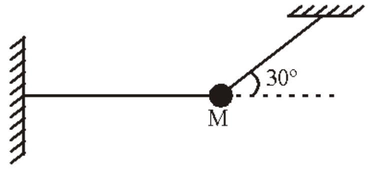
6. A mass *M* is hung with a light inextensible string. Tension in horizontal part of string is

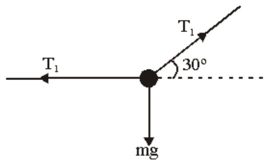


(a) $\sqrt{3}\ Mg$ (b) $\sqrt{2}\ Mg$

(c) $\frac{Mg}{\sqrt{3}}$ (d) $\frac{Mg}{2}$

Sol. (a) Here $T_1 = T_2 \cos 30 = T_2 \frac{\sqrt{3}}{2}$

i.e., $T_2 = \frac{2}{\sqrt{3}} T_1$



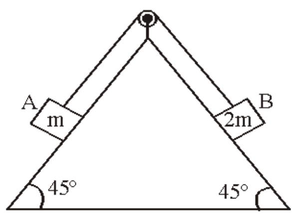
Also $Mg = T_2 \sin 30° = \frac{T_2}{2}$

i.e., $Mg = \frac{2}{\sqrt{3}} \frac{T_1}{2} = \frac{T_1}{\sqrt{3}}$

i.e., $T_1 = \sqrt{3}Mg$

---

7. Block *A* of mass *m* and block *B* of mass 2*m* are placed on a fixed triangular wedge by means of massless, inextensible string and a frictionless pulley as shown. The wedge is inclined at 45° to horizontal on both sides. The coefficient of friction between block *A* and wedge is 2/3 and that between block *B* and wedge is 1/3. If system of *A* and *B* is released from rest then acceleration of *A* is

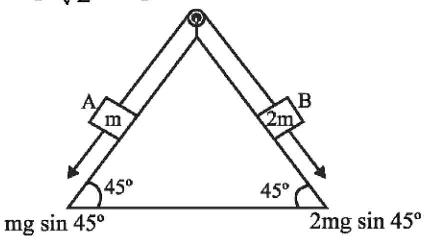


(a) zero (b) 1 ms^{-2}

(c) 2 ms^{-2} (c) 3 ms^{-2}

Sol. (a) f_{max} for $A = \mu_1 (mg\cos 45^\circ)$

$$= \frac{2}{3}\frac{mg}{\sqrt{2}} = \frac{\sqrt{2}}{3}mg$$



$F_2 = mg\sin 45^\circ$, $F_1 = 2mg\sin 45^\circ$

Also f_{max} for $B = \mu_2 (2mg\cos 45^\circ)$

$$= 1/3\left(2mg\sqrt{2}\right) - \frac{\sqrt{2}}{3}mg$$

Total frictional force

$$= \frac{\sqrt{2}}{3}mg + \frac{\sqrt{2}}{3}mg = \frac{2\sqrt{2}}{3}mg$$

But pulling force $= F_1 - F_2 = \frac{2mg}{\sqrt{2}} - \frac{mg}{\sqrt{2}} = \frac{mg}{\sqrt{2}}$

$\therefore$ system can not accelerate.

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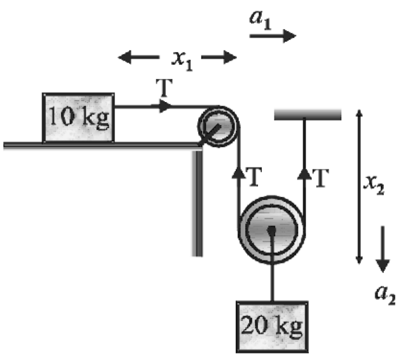
8. Two masses of 10 kg and 20 kg are connected with an inextensible string passing over two smooth pulleys as shown in the figure. The acceleration of the 20 kg mass will be



(a) 10 m/s^2 (b) $\frac{10}{3} m/s^2$

(c) $\frac{20}{3} m/s^2$ (d) $\frac{40}{9} m/s^2$

Sol. The 20 kg mass will move downwards such that T is the tension in the string. x_1 and x_2 is the length of horizontal portion and the length of each vertical string, respectively then the constraint relation is that the length L of the string is constant.

$x_1 + 2x_2 = L$

On differentiating twice, $\frac{d^2x_1}{dt^2} + \frac{2d^2x_2}{dt^2} = 0$

Or, $a_1 + 2a_2 = 0$, where a_1 and a_2 are respective accelerations of both the blocks

$a_1 = -2a_2$ or $|a_1| = 2|a_2|$

If $a_2 = a$, $a_1 = 2a$

The equations of the masses are given by

$T = 10$ ´ $2a = 20a$

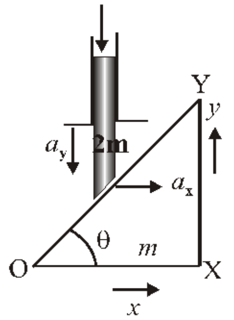
$20g - 2T = 20a$

$2T = 40a$

$$a = \frac{20g}{60} = \frac{20 \times 10}{60} = \frac{10}{3} \text{ m/s}^2$$

---

9. The figure below shows a rod of mass $2m$ restricted horizontally by two vertical supports that rest on a wedge of mass m. When the rod is released from rest, find the acceleration of the rod and that of the wedge.



(a) $\frac{2g}{2\tan\theta + \cot\theta}$ (b) $\frac{g}{2\tan\theta + \cot\theta}$

(c) $\frac{g}{\tan\theta + \cot\theta}$ (d) $\frac{2g}{\tan\theta + \cot\theta}$

Sol. Let mass 2m move down by y when wedge OXY moves towards the right through a distance x.

$\frac{y}{x}$ = tan θ, or $x = y \cot \theta$

Let the downward acceleration of 2*m* be a_y and the acceleration of *m* towards the right be a_x.

$\frac{d^2x}{dt^2} = a_x$, $\frac{d^2y}{dt^2} = a_y$,s

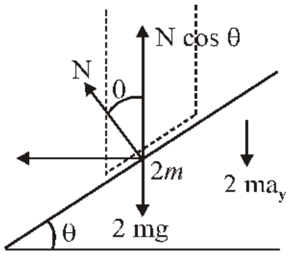
$a_x = \frac{d^2x}{dt^2} = \frac{d^2y}{dt^2} \times \cot\theta$

$a_x = a_y \cot\theta$

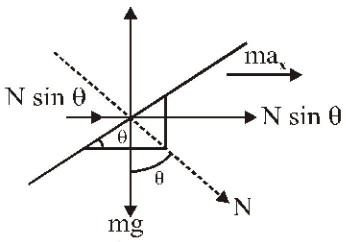
The free-body diagrams for mass of the rod and that of the wedge are shown in the figure below.

For the rod,

$2ma_y = 2mg - N\cos\theta$



(Rod)



(Wedge)

For the wedge,

$ma_y \cot\theta = ma_x = N\sin\theta$ (i)

For the rod,

$2mg - 2ma_y = N\cos\theta$ (ii)

On multiplying (i) by cos θ,

$N\sin\theta\cos\theta = ma_y \cot\theta\cos\theta$

On multiplying (ii) by sinθ,

$N\sin\theta\cos\theta = 2mg\sin\theta - 2ma_y\sin\theta$

On equating, we obtain

$ma_y \cot\theta\cos\theta + 2ma_y\sin\theta = 2mg\sin\theta$

$a_y(\cot\theta\cos\theta + 2\times\sin\theta) = 2g\sin\theta$

On dividing by sin θ,

$a_y\left[\cot\theta\frac{\cos\theta}{\sin\theta}+2\right] = 2g$

$a_y = \frac{2g}{2+\cot^2\theta}$

$a_x = a_y\cot\theta = \frac{2g\cot\theta}{2+\cot^2\theta} = \frac{2g}{2\tan\theta+\cot\theta}$

---

10. A chain of length *l* is placed on a smooth spherical surface of radius *R* with one of its ends fixed at the top of the sphere. Find the acceleration *a* of each element of the chain when its upper end is released. Assume that the length of the chain (*l*) is less than $\frac{\pi R}{2}$.

Sol. Consider an element *dl* of the chain at an angle θ to the vertical. If *m* is the mass of chain, the mass of element, $dm = \frac{m}{\ell} \times d\ell$.

$d\ell = Rd\theta$

$dm = \frac{m}{\ell}\times R\times d\theta$

dF = force responsible for acceleration = (*dm*)*g* sin θ

$dF = \frac{m}{\ell}\times R\times d\theta\times g\sin\theta$

Let α be the angle made by the chain with the radius vector *R*.

Net force = $F = \int dF = \frac{m}{\ell}\times R\times g\int_{\alpha}^{0}\sin\theta\, d\theta$

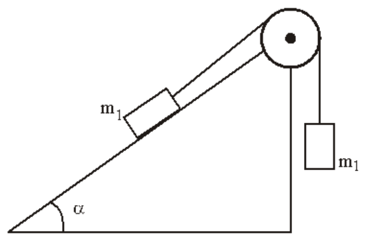
$= \frac{m}{\ell}\times R\times g\left|-\cos\theta\right|_0^{\alpha} = \frac{mRg}{\ell}[1-\cos\alpha]$

As $\alpha = \frac{\ell}{R}$,

$$F = \frac{mRg}{\ell}\left[1-\cos\left(\frac{\ell}{R}\right)\right]$$

$$\text{Acceleration} = \frac{F}{m} = \frac{Rg}{\ell}\left[1-\cos\left(\frac{\ell}{R}\right)\right]$$

11. The inclined plane of figure as shown below forms an angle α = 30° with the horizontal. The mass ratio m_2/m_1 = 2/3. The coefficient of friction between the body m_1 and the inclined plane is equal to μ =0.10. The masses of the pully and the threads are negligible. Find the magnitude and the direction of acceleration of the body m_2 when the formerly stationary system of masses starts moving.



Sol. According to the problem

$$\frac{m_2}{m_1} = \frac{2}{3} = 0.67 \quad \text{.... (i)}$$

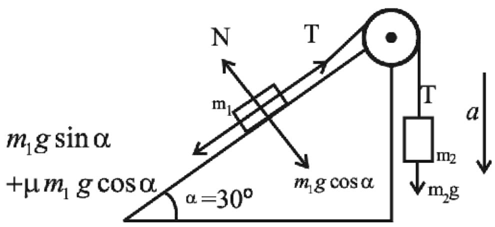
But

$$\mu\cos\alpha + \sin\alpha = 0.1\times\frac{\sqrt{3}}{2}+\frac{1}{2} = 0.5866 \ldots \text{(ii)}$$

From equation (i) and (ii), we get

$$\frac{m_2}{m_1} > \mu\cos\alpha + \sin\alpha$$

This condition satisfies the condition of problem. So, the acceleration of m_2 is in downward direction. (parallel to $\vec{g}$)



From free body diagram

$m_2g - T = m_2a$ ...... (iv)

$T - m_1g\sin\alpha - \mu m_1 g\cos\alpha = m_1a$ .... (v)

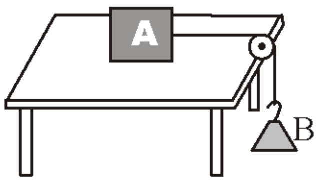
After solving equation (iv) and (v), we get

$$a = \frac{\left\{\left(\frac{m_2}{m_1}\right) - \sin\alpha - \mu\cos\alpha\right\}g}{1+\frac{m_2}{m_1}} = 0.05g$$

12. In figure, A and B are blocks with weights of 40N and 20N respectively.

(a) Determine the minimum mass of a block C be placed on A to keep it from sliding. The co-efficient of static friction between A and the table is μ_S = 0.20.

(b) Let the block C placed on A, be suddenly lifted off A. What is the acceleration of block A, if the co-efficient of kinetic friction between A and the table is μ_k = 0.15 ? (g = 10 ms^{-2}).



Sol. The driving force acting on (A + C) will be tension that equals the weight of B, i.e., 20N. The force of friction acting on A must balance it.

Thus, f = 20 newton

Now,

$$f \le \mu_S N \Rightarrow f \le 0.20(40\,\text{newton} + m\times 10)$$

$$\Rightarrow 20 \le 8 + 2m$$

$\Rightarrow m \le 6kg$. The minimum mass of block C is, thus, 6kg.

(b) When C is suddenly lifted off A, normal force falls to 40 newton, and hence limiting friction falls to 8N. But driving tension remains 20 N which starts motion. During motion kinetic friction starts acting and acceleration appears. The tension now changes. Let it be T_C. Then

$T - \mu_k m_A g = m_A a$ ..... (1)

$m_B g - T = m_B a$ ..... (2)

Solving these $a = \left(\dfrac{m_B g - \mu_k m_A g}{m_B + m_A}\right) = 1.4ms^{-2}$